Example: DC input

- For a DC input=0.3V
  - The output is expected to have period of 10 samples

<table>
<thead>
<tr>
<th>$V_Q$</th>
<th>$Y$</th>
<th>$q_E$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3</td>
<td>0</td>
<td>-0.3</td>
</tr>
<tr>
<td>0.6</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

Example: DC input, Multi-bit Quant.

- For a DC input=0.3V
  - The output is expected to have period of __ samples

<table>
<thead>
<tr>
<th>$V_Q$</th>
<th>$Y$</th>
<th>$q_E$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3</td>
<td>0.25</td>
<td>-0.3</td>
</tr>
</tbody>
</table>
Delta Modulators

- The goal was to achieve
  \[ Y(z) = (1 - z^{-1})q_E(z) \]

- Either single-bit or multi-bit quantizer can be used
  - The larger number of quantization levels, the better the performance (why?)

- This structure is also called “error-feedback modulator”

---

**SNR Improvement**

\[ S_{\text{out}}(f) = |H(e^{j\omega})|^2 S_{\text{in}}(f) \]

- If \( f_s = 1 \) (normalized)
  \[ S_{\text{out}}(f) = |2 \sin(\pi f)|^2 \frac{V_{\text{LSB}}^2}{12} \]

- If we assume \( f << 1 \)
  \[ |2 \sin(\pi f)|^2 \approx (2\pi f)^2 \Rightarrow S_{\text{out}}(f) = \pi f \frac{V_{\text{LSB}}^2}{3} \]

- The total in-band quantization noise
  \[ \sigma_{qE}^2 = \frac{1}{2 \cdot \text{OSR}} \int_{-1/(2 \cdot \text{OSR})}^{1/(2 \cdot \text{OSR})} (\pi f)^2 \frac{V_{\text{LSB}}^2}{3} \, df = \frac{2\pi^2 V_{\text{LSB}}^2}{3} \int_0^{1/(2 \cdot \text{OSR})} f^2 \, df \]

\[ \sigma_{qE}^2 = \frac{V_{\text{LSB}}^2}{12} \frac{\pi^2}{3 \cdot \text{OSR}^3} \]

---

University of Florida

N. Maghari
SNR Improvement

• Defining SQNR

\[
\text{SQNR} = \frac{P_{\text{sig}}}{P_{Q}}
\]

– For a sinewave with an amplitude of \( A \), \( P_{\text{sig}} = \frac{A^2}{2} \)
– Hence

\[
\text{SQNR} = \frac{P_{\text{sig}}}{P_{Q}} = \frac{18A^2OSR^3}{V_{\text{LSB}}^2\pi^2}
\]

– shows 9dB improvement per doubling the OSR
  • Example: \( V_{\text{LSB}} = 0.5 \), \( A = 1 \)
    @ OSR = 2 : \( 10\log\left(\frac{18 \times 2^3}{0.5^2 \pi^2}\right) = 17\text{dB} \)
    @ OSR = 4 : \( 10\log\left(\frac{18 \times 4^3}{0.5^2 \pi^2}\right) = 26\text{dB} \)

University of Florida

N. Maghari

Output Spectrum

– OSR=32, Q-levels=3

\[
\text{SNDR}= 51.961\text{dB}
\]

\[
\text{Normalized sampling frequency}
\]

\[
\text{Output}
\]
Random Quantization Error

- We assumed the PDF is uniform for the quantization error
  - In simulation of 1st order Δ Modulator

- Why $q_E$ error in Δ modulator is more random?